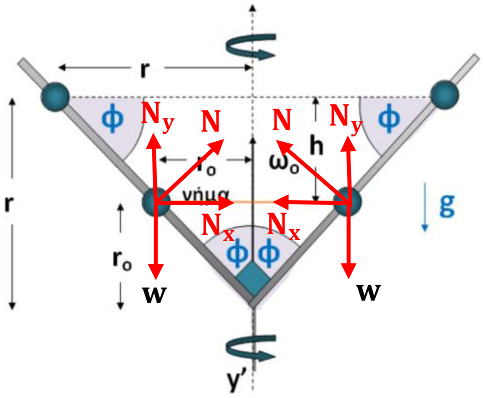
ΛΥΣΗ:

Δ1. Για τα σφαιρίδια ισχύει:

$$\Sigma F_y = 0 \Rightarrow N_y = w$$

(Ισχύει: $N_y = N\eta\mu\varphi$, $N_x = N\sigma\upsilon\nu\varphi$ και $N_x = N_y$)



$$\Sigma F_x = F_\kappa \Rightarrow N_x + T = m\omega_0^2 r_0 \Rightarrow w + T = m\omega_0^2 r_0 \Rightarrow$$
$$T = m(\omega_0^2 r_0 - g) \Rightarrow \boxed{T = 0}$$

Δ2.

$$\Sigma\tau_{\varepsilon\xi} = \frac{dL}{dt} = \frac{d}{dt}(2m\omega r_0^2) = 2mr_0^2\frac{d\omega}{dt} \Rightarrow$$
$$\Sigma\tau_{\varepsilon\xi} = 2mr_0^2\alpha_\gamma \Rightarrow \boxed{\Sigma\tau_{\varepsilon\xi} = 2{,}5\ \text{Nm}}$$

Δ3. $W_{F\varepsilon\xi} = \Delta K_{\sigma\upsilon\sigma} \Rightarrow W_{F\varepsilon\xi} = m\upsilon_1^2 - m\upsilon_0^2$ (1)

$$\upsilon_0 = \omega_0 r_0 \Rightarrow \upsilon_0 = 5\text{m/s}$$
$$\left.\begin{array}{r}\upsilon_1 = \omega_1 r_0\\ \omega_1 = \omega_0 + \alpha_\gamma t_1 = 4\text{rad/s}\end{array}\right\} \Rightarrow \upsilon_1 = 10\text{m/s}$$

Από τη σχέση (1):

$$\boxed{W_{F\varepsilon\xi} = 75\ \text{J}}$$

Δ4. Την $t_1 = 10\text{s}$ κόβεται το νήμα και $\alpha_\gamma = 0$

Από την Α.Δ.Μ.Ε.

$$E_{\alpha\rho\chi} = E_{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda} \Rightarrow \frac{1}{2}m\upsilon_1^2 + \frac{1}{2}m\upsilon_1^2 = \frac{1}{2}m\upsilon_2^2 + \frac{1}{2}m\upsilon_2^2 + 2mgh \Rightarrow$$
$$m\upsilon_1^2 = m\upsilon_2^2 + 2mgh \Rightarrow \upsilon_1^2 = \upsilon_2^2 + 2gh \Rightarrow \upsilon_1^2 = \omega^2 r^2 + 2g(r - r_0) \Rightarrow$$
$$100 = \omega^2 r^2 + 20(r - 2{,}5) \Rightarrow \omega^2 r^2 + 20r - 150 = 0 \quad (2)$$

Από την Αρχή Διατήρησης της Στροφορμής:

$$L_1 = L_2 \Rightarrow 2m\omega_1 r_0^2 = 2m\omega r^2 \Rightarrow 25 = \omega r^2 \Rightarrow 25\omega = \omega^2 r^2 \quad (3)$$

Από τη σχέση (2) παίρνουμε:

$$(2) \Rightarrow 25\omega + 20r - 150 = 0 \Rightarrow 5\omega + 4r = 30 \xRightarrow{\omega = \frac{25}{r^2}} 5\frac{25}{r^2} + 4r = 30 \Rightarrow \frac{125}{r^2} + 4r = 30 \Rightarrow$$
$$125 + 4r^3 = 30r^2 \Rightarrow 4r^3 - 30r^2 + 125 = 0$$

Από τη λύση της εξίσωσης προκύπτει:

$$r = \frac{5}{2}(1 + \sqrt{3}) \Rightarrow r = r_0(1 + \sqrt{3}) \Rightarrow r = r_0 + r_0\sqrt{3} \Rightarrow r - r_0 = r_0\sqrt{3} \Rightarrow \boxed{h = r_0\sqrt{3}}$$